Coherent-pulse implementations of quantum cryptography protocols resistant to photon-number-splitting attacks

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We propose a class of quantum cryptography protocols that are robust against photon-number-splitting attacks (PNS) in a weak coherent-pulse implementation. We give a quite exhaustive analysis of several eavesdropping attacks on these schemes. The honest parties (Alice and Bob) use present-day technology, in particular an attenuated laser as an approximation of a single-photon source. The idea of the protocols is to exploit the nonorthogonality of quantum states to decrease the information accessible to Eve due to the multiphoton pulses produced by the imperfect source. The distance at which the key distribution becomes insecure due to the PNS attack is significantly increased compared to the existing schemes. We also show that strong-pulse implementations, where a strong pulse is included as a reference, allow for key distribution robust against photon-number-splitting attacks.

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I. INTRODUCTION

Quantum cryptography, or more precisely, quantum key distribution (QKD) followed by the one-time pad, is the only physically secure way of transmitting secret information between the two authorized partners Alice and Bob [1–3]. Its security is not based on some mathematical assumptions, such as a limited eavesdropper's computational power, but on the laws of quantum mechanics. Alice prepares a quantum system in some state, encoding the information, and sends the system to Bob. The eavesdropper Eve cannot gain any knowledge about the quantum state without modifying the correlations between Alice and Bob, because, as it is well known, a measurement on an unknown quantum state normally modifies the state itself. Alternatively, the security of QKD schemes can be discussed in terms of the no-cloning theorem [4]: Eve cannot make and keep a perfect copy of the quantum state that Alice has sent to Bob [5].

Most of the known QKD protocols use two-dimensional quantum states, called qubits, as information carriers, although there exist alternative proposals using higher dimensional systems, either finite [6] or infinite [7]. The encoding of information can be performed by means of any two-dimensional quantum state, but very often this is done using photons because photons coupled in optical fibers (the *quantum channel*) propagate along large distances with almost no decoherence. Therefore, Alice must be able to prepare and send single photons to Bob: The existence of single-photon sources is then an implicit and crucial requirement for many of the proposed implementation of the existing schemes. There is a strong experimental effort in producing reliable single-photon sources, with remarkable achievements [8]. Because of their simplicity however, physicists often use sources that produce weak coherent pulses, $|\alpha\rangle=|\sqrt{\mu}e^{i\theta}\rangle$, with mean photon number $\mu\ll 1$, as an approximation of the single-photon pulse. Moreover, since there is no phase reference outside Alice's lab, the effective state used for the information encoding is

$$\rho=\int \frac{d\theta}{2\pi}|\sqrt{\mu}e^{i\theta}\rangle\langle\sqrt{\mu}e^{i\theta}|=\sum_n p(n,\mu)|n\rangle\langle n|, \qquad (1)$$

i.e., the mixture of coherent states with all possible phases is equivalent to a mixture of Fock states of n photons distributed according to a Poisson statistics of mean μ, $p(n,\mu)=e^{-\mu}\mu^n/n!$ [9,10]. Thus, a large fraction $p(0,\mu)$ of the pulses is empty; Alice produces the desired one-photon Fock state with probability $p(1,\mu)$; and, what is more problematic, Alice also produces multiphoton pulses with small but not negligible probability.

The fact that the presence of pulses with more than one photon may deteriorate the security of the protocol is intuitively clear: when a perfect copy of the quantum state is then produced, this copy could be kept by Eve, without introducing any error in the correlations Alice-Bob. Eve can then perform the so-called photon number splitting (PNS) attack that allows her to get information without being detected. Indeed, Lütkenhaus and Brassard *et al.* showed [10,11] that the presence of these multiphoton pulses makes the best-known QKD protocol, the BB84 scheme [1], insecure if the losses in the channel become important—that is, for long-distance implementations. This limits the distance up to which BB84 with weak coherent pulses and lossy optical fibers can be securely implemented. For typical experimental parameters this critical distance ℓ_c is of the order of 50 km. As we will show below, similar conclusions are valid for weak pulse implementations of other QKD schemes, such as the B92 [12] and the $4+2$ protocol [13]. The PNS attack is known to be ineffective against some QKD implementations that use entangled states (see for instance Ref. [3]). However, long-distance QKD with entangled photons is hard to implement. Therefore we focus on prepare-and-measure schemes (without entanglement).

Recently, quantum cryptography protocols have been proposed that are more robust against PNS attacks [14]. The scope of the present article is to give a detailed security analysis of these protocols under different eavesdropping scenarios. In Sec. II we review the PNS attack for the BB84 scheme, and we show how the same results and conclusions also apply for the B92 and $4+2$ protocols. Then, we discuss QKD implementations including a strong reference pulse as

a first possibility for minimizing the importance of PNS attacks. The results of this section give the necessary insight to construct the new protocols that are more resistant to PNS attacks. These are presented in Sec. III. We will focus on a particular one, that differs from BB84 only in the classical sifting procedure. We will consider various possible attacks, some which do not introduce errors, some which use cloning machines (which do introduce some errors), and some which are the combination of both. We briefly discuss the experimental data of Ref. [15] in the light of our results, as an example of a QKD implementation secure against the considered PNS attacks. In Sec. IV we explore possible generalizations using a larger number of states. The last section summarizes the main results.

Once the contents of the paper are settled, it is also important to stress that the present work is a preliminary investigation—note that the BB84 protocol has been the object of intensive studies during more than a decade. That is why we work under several simplifying assumptions, that allow a simple discussion of the advantages of the new protocols, leaving for further investigation the task of possibly relaxing them. The main assumptions are as follows:

(i) The comparison between the new protocols and the BB84 is made for a constant value of μ; specifically, we take $\mu=0.1$ for BB84, and we adapt μ for the other protocols in order to have the same raw rate. Ideally, the comparison should be done by choosing the optimal value of μ at any distance, for each protocol.

(ii) We do not take into account collective attacks, where Eve interacts coherently with more than one pulse. In the type of PNS attacks considered in this work, Eve can measure the number of photons in each pulse, keep some photons in a quantum memory until the basis reconciliation, and replace the lossy line by a lossless line. Moreover, we assume that she measures the kept photons before Alice and Bob start any error correction and privacy amplification process [16].

(iii) We do not consider advantage distillation protocols for secret-key distillation (see, for instance, Ref. [17]). Therefore, a protocol is said secure if and only if the information Alice-Bob is greater than Eve's information. Indeed, it was shown in Ref. [18] that secret-key distillation is possible using one-way privacy amplification whenever

$$I_{AB}>\min(I_{AE},I_{BE}). \tag{2}$$

(iv) Moreover, the imperfections of the detectors (reduced quantum efficiency $\eta_d<1$, dark counts...) will be taken into account only in Sec. IV. The first comparison of the BB84 protocol with the new one (Sec. III) will be done for perfect detectors.

II. PNS ATTACK

Any experimental realization using photons of a QKD protocol with two-dimensional quantum states must ideally be performed with a single-photon source. Unfortunately, this is a very strong requirement with present-day technology, and one has to design a way of experimentally approximating the single-photon source. In spite of the fact that QKD has proven to be unconditionally secure (see, for instance, Ref. [19]), this may not be the case any longer if the technology of the honest parties is not perfect.

In most of the existing implementations, the one-photon pulse is approximated by a weak coherent pulse $|\sqrt{\mu}e^{i\theta}\rangle$. As said above, since there is no absolute phase reference, the state seen by Bob and Eve is given by Eq. (1), an incoherent mixture of n-photon states with Poisson probabilities. Eve can then perform a photon number nondemolition measurement, keep one of the photons when a multiphoton state is found, and forward the rest to Bob. Note that Eve's action is not detected by Bob if he is assumed to have only access to the average detection rate, and not to the statistics of the photons he receives. We also assume that Eve is able to control the losses on the line connecting Alice and Bob (or equivalently she can send photons to Bob by a lossless line). In this situation, Eve can perform the so-called PNS attack that, as we show below, limits the security of many of the known existing protocols.

A. BB84 protocol

In the BB84 protocol [1], Alice chooses at random between two mutually unbiased bases, in which she encodes a classical bit. Denoting by $|\pm x\rangle$ ($|\pm y\rangle$) the eigenvectors of σ_x (σ_y) with eigenvalue ±1, she can encode a logical 0 into either $|+x\rangle$ or $|+y\rangle$ and a 1 into either $|-x\rangle$ or $|-y\rangle$. She sends the qubit to Bob, who measures at random in the x or y basis. Then, they compare the basis and when they coincide, the bit is accepted. In this way, half of the symbols are rejected, and, in the absence of perturbations, Alice and Bob end up with a shared secret key. In practical situations, and due to the presence of errors and possibly a spy, some error correction and privacy amplification techniques have to be applied, in order to extract a shorter completely secure key.

Now, let us see how Eve can take advantage of the multiphoton pulses. Alice sends a pulse with $\mu\ll1$ coding the classical bit (say on light polarization). Eve performs the photon number measurement and when two or more photons are detected, she takes one and forwards the rest to Bob by her lossless line. Eve stores the photon in a quantum memory and waits until the basis reconciliation. Once the basis is announced, she has only to distinguish between two orthogonal states, which can be done deterministically. Thus, for all the multiphoton pulses Eve obtains all the information about the sent bit. If Alice and Bob are in principle connected by a lossy line, Eve can block some of the single-photon pulses, and forward the multiphoton pulses, on which she can obtain the whole information, by her lossless line. In this way, Alice and Bob do not notice any change in the expected raw rate, and Eve remains undetected. When the losses are such that Eve can block all the single-photon pulses, the protocol ceases to be secure.

Denote by α the losses [dB/km] on the line. The transmission on a line of length ℓ [km] is

$$t=10^{-\delta/10},\quad \delta=\alpha\ell. \tag{3}$$

As we said, we keep the discussion simple by considering the case of perfect detectors: anyway, PNS attacks on the

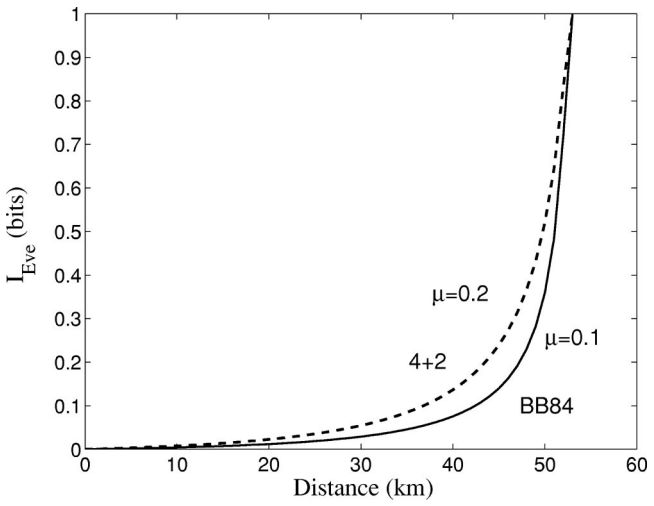

FIG. 1. Eve's information as a function of the distance for the PNS attacks described in the text.

BB84 protocol have been thoroughly studied in Refs. [10] and [11]. If the detectors are perfect, Bob counts a photon whenever he receives at least one, so the raw detection rate per pulse is simply

$$R_{\mathrm{Bob}}=\sum_{n\geqslant 1} p(n,\mu t)=1-p(0,\mu t). \tag{4}$$

Eve is placed just outside Alice's lab, and is supposed to apply only the PNS attack. Whenever Alice produces more that one photon, Eve can keep one, since she forwards the rest on a perfect line to Bob, who anyway will detect something. The only constraint that Eve must fulfill to be undetected is that the raw rate must not change; to ensure this, Eve should let a fraction q of the one-photon pulses go to Bob, in such a way that

$$R_{\mathrm{Bob}}^{\mathrm{PNS}}=qp(1,\mu)+\sum_{n\geqslant 2} p(n,\mu) \tag{5}$$

is equal to R_{Bob}. If the losses t are such that q can be zero in Eq. (5), that is, when all the one-photon pulses can be blocked, then Eve gets all the information, without being detected: Eve's information, in percent of the length of the key, is

$$I_{\mathrm{Eve}}=\frac{1}{R_{\mathrm{Bob}}}\sum_{n\geqslant 2} p(n,\mu)\equiv\frac{R_{\mathrm{BB84}}}{R_{\mathrm{Bob}}}. \tag{6}$$

The critical attenuation δ_c at which Eve knows all the key using the PNS attacks is then given by the condition $R_{\mathrm{BB84}}=R_{\mathrm{Bob}}$. In Fig. 1 we show the variation of I_{Eve} as a function of ℓ for $\mu=0.1$ and $\alpha=0.25$ dB/km [20]. The critical attenuation in this case is $\delta_c=13$ dB, and the corresponding distance $\ell_c=52$ km.

Just a remark to say that this value for the distance is not significantly modified if one takes into account imperfect detectors, provided that Eve cannot improve the performances of these detectors. The argument goes as follows: Bob receives almost always one photon, both in the absence of Eve because $p(1,\mu t)\gg p(2,\mu t)$ and in its presence because $p(2,\mu)\gg p(3,\mu)$. Consequently, the constraint reads $\eta_d R_{\mathrm{Bob}}=\eta_d^{\mathrm{PNS}}R_{\mathrm{Bob}}^{\mathrm{PNS}}$. If Eve cannot modify the detectors' efficiency, $\eta_d^{\mathrm{PNS}}=\eta_d$ and the distance at which $q=0$ is independent of this efficiency. Conversely, if Eve can modify Bob's detection so that $\eta_d^{\mathrm{PNS}}=1$, this is obviously an advantage for her: For instance, if $\eta_d=0.1$, δ_c would be reduced by 10 dB, that is, ℓ_c will be reduced by 40 km. Indeed, a ℓ_c of some 10 km has been announced in Ref. [11], where Eve's possibility of modifying Bob's detectors was taken into account. In our opinion however, it is unreasonable to allow Eve entering Bob's lab to modify his detectors, basically because if Eve can modify Bob's detectors, there is no reason why she cannot also have put an emitter in Bob's computer and simply read his data [21].

One may wonder whether the PNS attack is possible only because the information is encoded on light polarization. This is not the case: The same reasoning is also valid for other encodings such as, for instance, in the time-bin scheme (see Ref. [3]) where the information is transmitted using the relative phase between two weak coherent pulses that are sent through the fiber. In principle, the state leaving Alice's side is $|\phi\rangle=|\sqrt{\mu}e^{i\theta}\rangle|\sqrt{\mu}e^{i\theta}e^{i\phi}\rangle$ where $\phi=0$, π ($\phi=\pm\pi/2$) correspond to $\pm x$ ($\pm y$). But since there is no phase reference, the effective state seen by Eve and Bob is again

$$\rho=\int\frac{d\theta}{2\pi}|\phi\rangle\langle\phi|=\sum_n p(n,2\mu)|\varphi_n(\phi)\rangle\langle\varphi_n(\phi)|, \tag{7}$$

where $p(n,2\mu)$ are Poisson probabilities of mean photon number 2μ and

$$|\varphi_n(\phi)\rangle=\sum_{m=0}^{n}\sqrt{\binom{n}{m}\frac{1}{2^n}}e^{im\phi}|n-m\rangle|m\rangle. \tag{8}$$

Note that Bob's state is given by an expression like Eq. (7) multiplying the mean photon number by the channel attenuation. It is possible to define a creation and annihilation operator

$$a^{\dagger}(\phi)=\frac{a_1^{\dagger}+e^{i\phi}a_2^{\dagger}}{\sqrt{2}},$$
$$a(\phi)=\frac{a_1+e^{-i\phi}a_2}{\sqrt{2}}, \tag{9}$$

such that acting on the two-mode vacuum state gives $a^{\dagger}(\phi)|0,0\rangle=|\varphi_1(\phi)\rangle$. It is straightforward to see that

$$|\varphi_n(\phi)\rangle=\frac{[a^{\dagger}(\phi)]^n}{\sqrt{n!}}|0,0\rangle, \tag{10}$$

$[a^{\dagger},a]=1$ and $\langle\varphi_{n'}(\phi)|\varphi_n(\phi)\rangle=\delta_{n,n'}$. Thus, the situation is the same as in the previous polarization encoding scheme [10]. Eve can count the total number of photons in the two (now temporal) modes, in an analogous way as in the previous photon number measurement for polarization, without

being noticed by Bob. When ''more than one'' photons are detected, i.e., she projects into $|\varphi_2\rangle$, she stores one copy of the state in her quantum memory until the basis reconciliation. Obviously, the equations and critical values in this case are the same as the ones found above for the polarization encoding scheme.

B. B92 protocol

An alternative QKD scheme is given by the B92 protocol [12]. The classical bit is simply encoded by Alice using two nonorthogonal states, $|\psi_0\rangle$ and $|\psi_1\rangle$ with $\langle\psi_0|\psi_1\rangle\neq 0$. Without losing generality we take [22]

$$|\psi_0\rangle=\begin{pmatrix}\cos\frac{\eta}{2}\\ \sin\frac{\eta}{2}\end{pmatrix} \quad |\psi_1\rangle=\begin{pmatrix}\cos\frac{\eta}{2}\\ -\sin\frac{\eta}{2}\end{pmatrix}, \tag{11}$$

with $0\leqslant\eta\leqslant\pi/2$ and the overlap is $|\langle\psi_0|\psi_1\rangle|=\cos\eta$.

Bob has to distinguish between two nonorthogonal quantum states, and this can only be done with some probability. The measurement optimizing this probability is defined by the following positive operators, summing up to one [23]:

$$\Pi_0=\frac{1}{1+\cos\eta}|\psi_1^\perp\rangle\langle\psi_1^\perp|,$$

$$\Pi_1=\frac{1}{1+\cos\eta}|\psi_0^\perp\rangle\langle\psi_0^\perp|, \tag{12}$$

$$\Pi_?=\mathbb{1}-\Pi_0-\Pi_1,$$

where $|\psi^\perp\rangle$ denotes the state orthogonal to $|\psi\rangle$. When Bob's measurement outcome is the one associated to Π_i, with $i=0$, 1, he knows that the state was $|\psi_i\rangle$. The probability of obtaining an inconclusive result is equal to the overlap between the states, $p_?=\langle\psi_0|\Pi_?|\psi_0\rangle=\langle\psi_1|\Pi_?|\psi_1\rangle=|\langle\psi_0|\psi_1\rangle|=\cos\eta$. Thus, Alice and Bob will accept the sent bit only for those cases where Bob's measurement gives a conclusive result. The probability of acceptance is $p_{ok}=1-\cos\eta$, while for the BB84 this probability is equal to one half. Eve's PNS attack is described in the following lines.

In a weak pulse encoding scheme, this protocol is clearly insecure. What Eve can simply do is to perform the same unambiguous measurement as Bob. When a conclusive result is found, she knows the state and she prepares a copy of it on Bob's side. When Eve is not able to determine the state, she blocks the pulse. Of course, as soon as we have some losses in the channel Alice and Bob cannot detect the eavesdropping (since they assume that the absence of signal is due to the losses), and the protocol is insecure. Note that in this case Eve does not need any quantum memory and lossless line.

C. 4+2 protocol

A third QKD protocol was proposed in Ref. [13] combining some of the ideas of the B92 and BB84 schemes. As in the BB84 protocol, there are four states grouped into two sets

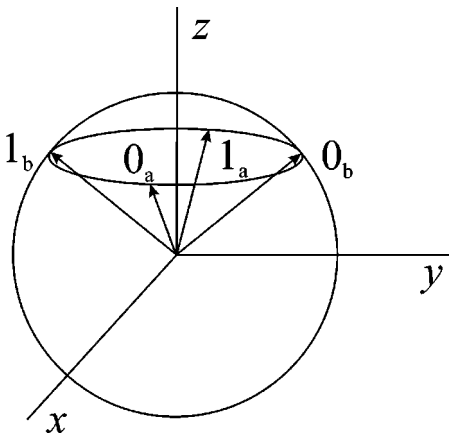

FIG. 2. Set of states needed for the 4+2 protocol.

$\{|0_a\rangle,|1_a\rangle\}$, $\{|0_b\rangle,|1_b\rangle\}$. However, as in the B92, the states in each set are not orthogonal, their overlaps being $|\langle 0_a|1_a\rangle|=|\langle 0_b|1_b\rangle|=\cos\eta$. The situation is depicted in Fig. 2, the four states lie on the same parallel of the Bloch sphere. Thus, Alice chooses randomly in which of the two sets the bit is encoded. Bob performs at random one of the two (POVMs) distinguishing the two states of each set. After basis reconciliation, they determine all the cases where Bob has applied the correct measurement obtaining a conclusive result. At first sight, this protocol seems more resistant against PNS attacks: compared to the BB84 case, Eve can keep some of the photons but her measurement after the basis reconciliation may not be conclusive. Compared to the B92 case, she does not know which of the two measurements has to be applied. However, and due to the particular geometry of the sets of states, this scheme does not offer any advantage over the two previous ones. But before describing Eve's attack, let us show how the three-outcome POVM described by Eq. (12) can be interpreted as the concatenation of two two-outcome measurements.

The effect of any quantum measurement can be represented by a set of operators $\{A_i\}$ satisfying $\Sigma_i A_i A_i^\dagger=\mathbb{1}$. If the initial state is ρ, the probability for any outcome, say i, is

$$p_i=\mathrm{tr}(A_i\rho A_i^\dagger), \tag{13}$$

and the state is transformed into

$$\rho_i=\frac{1}{p_i}A_i\rho A_i^\dagger. \tag{14}$$

Consider the states (11). The POVM described by the operators (12) can be effectively replaced by a sequence of two two-outcome measurements. First, one applies a measurement described by the operators

$$A_{ok}\equiv\frac{1}{\sqrt{1+\cos\eta}}(|+x\rangle\langle\psi_1^\perp|+|-x\rangle\langle\psi_0^\perp|)$$

$$A_?\equiv\sqrt{\mathbb{1}-A_{ok}A_{ok}^\dagger}. \tag{15}$$

The effect of this first measurement is the following: With probability $p_{ok}=1-\cos\eta$ the state $|\psi_0\rangle$ ($|\psi_1\rangle$) is mapped into $|+x\rangle$ ($|-x\rangle$). This operation is often called a filtering, and it is equivalent to the cases where the POVM (12) gives a conclusive result. When the outcome *ok* has been obtained, it is said that the states have passed the filter. If this is the case, a standard von Neumann measurement on the x basis suffices for discriminating between the two states.

Let us come back to the $4+2$ protocol and consider the filter for the states in set a, sending these states into the x basis. It is not difficult to see that the same filter maps the states in set b into $|\pm y\rangle$. Therefore, a BB84-like situation is recovered.

It is now easy to design a PNS attack. First, Eve counts the number of photons. Similar to the B92 case, she applies the filtering two-outcome measurement when a multiphoton pulse is obtained. When the result is conclusive, she keeps the resulting photon in a quantum memory and forwards the rest of the photons to Bob. Then, as in the BB84 case, she waits for the basis reconciliation, and performs the right von Neumann measurement allowing her to read the bit. In order to make a fair comparison, we always impose the same key rate in the absence of Eve as in BB84 using $\mu_{\mathrm{BB84}}=0.1$. In this case we must have

$$\mu_{4+2}=\mu_{\mathrm{BB84}}/(1-\cos\eta). \tag{16}$$

In a similar way as above for the BB84 case, one can compute Eve's information for this attack. It almost coincides with the curve found for the BB84 protocol, and the critical distance is again $\ell_c=52$ km (see Fig. 1). Indeed, the critical distance turns out to be quite independent of the degree of nonorthogonality between the states in the $4+2$ protocol, if one imposes the equality of the raw rates (16).

The analysis of the $4+2$ protocol ends the present section. All the studied QKD schemes do not guarantee a secure key distillation when the channel attenuation is around 15 dB. Unfortunately, the use of nonorthogonal states has not been enough for avoiding Eve's attacks. The critical distance basically corresponds to the point where the raw rate on Bob's side can be simulated by the number of multiphoton pulses leaving Alice's lab.

D. Strong pulse implementations

The three protocols analyzed in the previous sections are not robust against PNS attacks in a weak coherent pulse implementation. Eve exploits the presence of multiphoton pulses and the losses on the line. At the critical distance, the losses allow her to block all the pulses for which her attack has not succeeded, without being noticed. A possible way of avoiding this problem is to send also a strong reference pulse that *must always be detected* on Bob's side, as in the original B92 proposal [12]. In this way, Eve cannot block the pulses without introducing errors. This modification is rather easy to handle also at the level of the hardware: one just needs to add and monitor a new detector that checks the presence of the strong pulse. In the following lines we consider these implementations from the point of view of PNS attacks.

The information encoding uses the relative phase between a weak coherent pulse with respect to a strong reference pulse that is sent later through the line. Thus, Alice prepares a weak coherent pulse and a strong pulse, $|\phi\rangle=|\sqrt{\mu'}e^{i\theta}\rangle|\sqrt{\mu}e^{i\theta}e^{i\phi}\rangle$, where $\mu'\gg\mu$ and $\phi=0,\pi$ encodes the classical bit. This is obviously a realization of a B92 scheme, since $|\langle 0|\pi\rangle|=e^{-2\mu}\neq 0$; the analogous scheme using two sets of states, $\phi=0,\pi$ for one of the sets and $\phi=\pm\pi/2$ for the other, is an implementation of the $4+2$ scheme. Let us focus on the B92 (as we will see, the same conclusions are valid for the other schemes). Denote by r the ratio between the two intensities $r=\mu/\mu'\ll 1$. Bob delays the weak pulse and makes it interfere with a fraction r of the strong pulse. Constructive and destructive interference correspond to the values 0 and π. The probability of inconclusive results is $p_?=e^{-2\mu}$ as expected (see Ref. [24] for a practical implementation of this measurement), and the transmission rate for small μ is $\sim 2\mu$ [13]. The detection of the $1-r\lesssim 1$ fraction of the strong reference pulse by Bob should allow him to detect Eve's intervention, i.e., none of the pulses can be blocked. In particular, Eve cannot limit herself to forward photons only when she has obtained a conclusive result for the unambiguous measurement. Note that this forces the strong-pulse mean photon number to be significant at Bob's side.

Of course, Eve can always take advantage of the multiphoton pulses for acquiring partial information, even if not full information. Here is the analysis of the PNS attack in the present implementation. Since as usual there is no global phase reference available, the effective state leaving Alice's lab is

$$\rho=\int\frac{d\theta}{2\pi}|\phi\rangle\langle\phi|=\sum_n p(n,\mu+\mu')|\varphi_n(\phi)\rangle\langle\varphi_n(\phi)|, \tag{17}$$

where $p(n,\mu+\mu')$ are Poisson probabilities and

$$|\varphi_n(\phi)\rangle=\sum_{m=0}^{n}\sqrt{\binom{n}{m}\frac{r^m}{(1+r)^n}}e^{im\phi}|n-m\rangle|m\rangle. \tag{18}$$

In a similar way as above, one can define

$$a^\dagger(\phi)=\frac{1}{\sqrt{1+r}}(a_1^\dagger+\sqrt{r}e^{i\phi}a_2^\dagger)$$

$$a(\phi)=\frac{1}{\sqrt{1+r}}(a_1+\sqrt{r}e^{-i\phi}a_2), \tag{19}$$

such that acting on the two-mode vacuum state gives $a^\dagger(\phi)|0,0\rangle=|\varphi_1(\phi)\rangle$. Again, we have

$$|\varphi_n(\phi)\rangle=\frac{[a^\dagger(\phi)]^n}{\sqrt{n!}}|0,0\rangle, \tag{20}$$

$[a^\dagger,a]=1$ and $\langle\varphi_{n'}(\phi)|\varphi_n(\phi)\rangle=\delta_{n,n'}$. Eve can perform a nondemolition measurement for these number states without

being detected by Bob. Indeed, his state is the same as in Eq. (17), just taking into account the channel attenuation.

Denote the channel losses by δ. Since $\mu' \gg \mu$, Eve's Poisson distribution is centered around μ' while Bob's around $\mu' t = \mu' 10^{-\delta/10}$. Moreover the strong pulse must be always detected by Bob, so we will impose $\mu' 10^{-\delta/10} = 10$ (at least), which means that $\mu' = 10^{(1+\delta/10)}$. In order to make a fair comparison with the BB84 scheme using $\mu = 0.1$, we take the same raw rate in the absence of Eve at the critical distance, which leads to

$$\frac{\mu_{\mathrm{BB84}}}{2} = 2\mu_{\mathrm{B92}} \tag{21}$$

and then $\mu_{\mathrm{B92}} = 0.025$, i.e., $|\langle 0|\pi\rangle| = 0.95$, and $r = 10^{-(2+\delta/10)}/4$.

Now, Eve performs the measurement in the $|\varphi_n\rangle$ basis. Since her Poisson probability is centered around μ', she obtains a pulse containing (on average) μ' photons. On Bob's side a pulse with ten photons is expected, so Eve keeps $|\varphi_{\mu'-10}\rangle$ and forwards $|\varphi_{10}\rangle$ to Bob by her lossless line. Eve's intervention remains unnoticed to Bob. Eve is now faced with the problem of detecting two states having an overlap

$$|\langle \varphi_{\mu'-10}(\pi)|\varphi_{\mu'-10}(0)\rangle| = \left(\frac{1-r}{1+r}\right)^{\mu'-10} \sim \left(\frac{1-r}{1+r}\right)^{\mu'}. \tag{22}$$

She applies the measurement maximizing her information [25], obtaining

$$I_{\mathrm{Eve}} = I(p_e), \tag{23}$$

where $I(p) = 1 + \log_2 p + (1-p)\log_2(1-p)$ is the binary mutual information (in bits) and p_e is the error probability,

$$p_e = \frac{1}{2}\left(1 - \sqrt{1 - |\langle \varphi_{\mu'-10}(\pi)|\varphi_{\mu'-10}(0)\rangle|^2}\right). \tag{24}$$

It is not hard to compute the limit for Eve's information. For very large distances, $\mu' \to \infty$ and then

$$|\langle \varphi_{\mu'}(\pi)|\varphi_{\mu'}(0)\rangle| = \lim_{\mu'\to\infty}\left(\frac{1-\mu/\mu'}{1+\mu/\mu'}\right)^{\mu'} = e^{-2\mu}, \tag{25}$$

i.e., the initial overlap gives the searched limit and $I_{\mathrm{Eve}} \sim 0.07$ bits. Thus, for any distance, the protocol is clearly secure against PNS attacks. The same is valid for the strong pulse realization of the BB84 protocol, which, as said, is the $4+2$ scheme.

Note that strong pulse implementations appear as an intermediate step in the transition from discrete to continuous variables QKD schemes using coherent states [7]. There, a strong reference pulse, with a very large mean photon-number μ', is sent through the channel with a weaker pulse, containing about hundred photons. The security comes from the fact that although μ is not weak, an infinite range of values is used for the information encoding (while, for example, we have only two in the B92 case) and Eve is not able to discriminate which state has been sent. Nevertheless, many of the results presented in this section can be translated to these protocols, opening the possibility of new eavesdropping attacks.

An important point about strong pulse QKD implementations was somehow hidden in the previous discussion. As said, one must ensure a reasonable photon number for the strong pulse on Bob's side, i.e., the condition $\mu' 10^{-\delta/10} - 10$ must be always satisfied. Therefore, μ' should be increased with the distance Alice-Bob, while μ is fixed by the desired overlap between the two states used in the B92 scheme, independently of the distance. In the previous lines we took a quite conservative value, coming from Eq. (21). We can indeed consider $\mu = 1/4$, which gives $|\langle 0|\pi\rangle| = 0.6$ and $I_{\mathrm{Eve}} \sim 0.5$. This forces μ' and the ratio r to increase with the distance, which can lead to problems in the interferometric arrangement needed for detection. For instance for a distance of 80 km, that taking as usual $\alpha = 0.25$ means 20 dB, we have $\mu' = 10^3$ and $r = 10^{-4}/4$. However if these requirements are met, a secure implementation becomes possible with a key generation rate significantly larger than for the BB84 scheme using $\mu = 0.1$.

For the rest of the paper however, we will not consider this type of scenario and we will only deal with implementations using weak coherent pulses.

III. QKD PROTOCOLS RESISTANT TO PNS ATTACKS

The aim of the present section is to give QKD protocols resistant to the PNS attack in a weak pulse implementation. From the previous discussion we can understand some of the basic requirement for these schemes. We have seen above that the $4+2$ protocol was as vulnerable as B92 against PNS attacks because, in spite of using two sets of states instead of one, a single quantum operation (15) allows Eve to make pairwise orthogonal the states in the sets *a* and *b*. After successfully performing this operation, she can wait for the basis reconciliation, as in the BB84 case, and read the information by a von Neumann measurement. Alice can encode her information into pairs of nonorthogonal states belonging to different sets; but, to increase the robustness against PNS attacks, she must also choose these sets carefully: No quantum operation should exist that increases, even probabilistically, the overlap of the states in all sets at the same time.

A simple choice of such sets is as follows: One takes the two sets of the $4+2$ scheme and reflects one of them with respect to the *xy* plane (see Fig. 2). Other solutions are available that are simpler to visualize: Actually, one can restrict oneself to any plane in the Bloch sphere, as in the BB84 case. This situation is depicted in Fig. 3. The general expression for these states is

$$|0_a\rangle = \begin{pmatrix} \cos\frac{\eta}{2} \\ \sin\frac{\eta}{2} \end{pmatrix} \quad |1_a\rangle = \begin{pmatrix} \cos\frac{\eta}{2} \\ -\sin\frac{\eta}{2} \end{pmatrix}$$

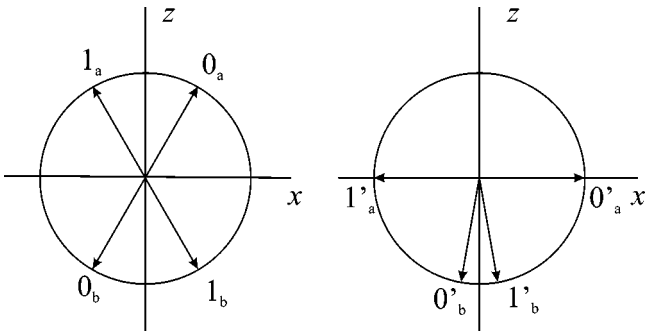

FIG. 3. States configuration for a QKD protocol robust to PNS attacks.

$$|0_b\rangle = \begin{pmatrix} \sin\frac{\eta}{2} \\ -\cos\frac{\eta}{2} \end{pmatrix} \quad |1_b\rangle = \begin{pmatrix} \sin\frac{\eta}{2} \\ \cos\frac{\eta}{2} \end{pmatrix}. \tag{26}$$

After successful application of the filter that makes the states in set *a* orthogonal, the overlap between the states in set *b* has significantly increased. Indeed, it can be shown that no quantum operation can decrease the overlap of the states in both sets *a* and *b* (see Appendix A). So, now Eve has to consider two different filters F_a and F_b that make the states in set *a* and set *b* orthogonal, respectively. If she wants to get the whole information about the bit sent by Alice, she has to block all the pulses with less than three photons. When the pulse contains three photons, she applies F_a to the first one, F_b to the second one, and only when both of them are conclusive, she forwards the third photon to Bob. It is clear that the distance of Alice-Bob, such that Eve can perform this attack without being detected, is much larger than above. It basically corresponds to the point where the raw rate is equal to the number of pulses on Alice's side with more than two photons.

Using this idea, we can design different state configurations. One of them turns out to be equivalent, at the quantum level, to the BB84 scheme. The states and the measurements are the same as in this protocol, the only difference being in the reconciliation process. But, surprisingly, this variation makes the protocol significantly more resistant to PNS attacks. The remaining of this section will be devoted to the detailed security analysis of this protocol, that was first proposed in Ref. [14].

A. Four-state protocol

The configuration of states in Fig. 3 allows Alice and Bob to exchange a key in a secure way for larger distance than for many of the existing protocols. In the case in which the angle between the states in each set is $\pi/2$ we recover a BB84-like state configuration. Nevertheless, note that Alice's bit encoding has radically changed (see Fig. 3), since orthogonal states encode the same classical bit.

Like we did for BB84, we suppose that Alice uses as information carriers the eigenvectors of σ_x and σ_y. Now, the bit 0 is encoded into $|\pm x\rangle$ and 1 into $|\pm y\rangle$. Consider the case in which Alice's bit is equal to zero. She chooses randomly between $|\pm x\rangle$ and sends the state, say $|+x\rangle$, to Bob. Bob measures randomly in the *x* or *y* basis. After this, Alice starts the reconciliation process announcing the sent state and one of the two possible states encoding one, for instance $\{|+x\rangle,|+y\rangle\}$. If Bob's measurement was in the *x* basis, the result was $+1$ (remember that the sent state was $|+x\rangle$). This result would also have been possible if Alice had sent the other state she declared, here $|+y\rangle$, so Bob cannot discriminate between the two alternatives. If Bob measured in the *y* basis, for half of the cases the result was $+1$ and for the rest -1. In the first case, again he cannot discriminate; but in the latter, he knows for sure that the sent bit was not $|+y\rangle$, so it must have been $|+x\rangle$: Bob accepts the bit 0. At first sight this is just a trivial and artificially complicated modification of the BB84 protocol. However with these variations the obtained protocol is much more resistant to Eve's attacks.

Eve is faced with the following problem: After Alice's announcement she will have to deal with one of four possible sets of two states:

$$\begin{aligned} s_1 &\equiv \{+x,+y\} \quad s_2 \equiv \{+y,-x\}, \\ s_3 &\equiv \{-x,-y\} \quad s_4 \equiv \{-y,+x\}. \end{aligned} \tag{27}$$

Eve can determine the sent state unambiguously, with some probability, when the pulse contains at least three photons. Indeed she measures in the *x* and *y* basis the two first photons, which allows her to discard two of the four possibilities. Then, she applies to the third photon the measurement discriminating between the two remaining states. This intuitively shows that this scheme is more robust against PNS attacks, since only three-photon pulses provide her with the full information. In the next lines we will extend these ideas in a more precise way, showing that the distance for a secure implementation of this protocol is approximately twice the one for the standard BB84, once the value of μ is fixed according to the rule we follow in this paper [see remark (i) at the end of Sec. I].

A new protocol requires the analysis of a full set of attacks by Eve, some of which may be new ones. In Sec. III B, we deal with attacks exploiting the presence of multiphoton pulses without introducing errors on Bob's side. These are the typical PNS attacks, that motivated the discovery of this protocol. In Sec. III C, we change completely our standpoint: We suppose that we have single-photon sources, and we study individual attacks based on cloning machines. Surprisingly, it turns out that the new protocol is better than the BB84 also on this ground, although the improvement is very small. Finally, in Sec. III D we combine PNS and cloning attacks in a kind of eavesdropping strategy that has never been considered before.

B. PNS attacks

The first type of attacks we consider are of the same type as the PNS attack for the BB84. Eve does not introduce any error on Bob's side, she just uses the multiphoton pulses for acquiring information.

Let us first calculate the *critical distance* at which Eve can obtain full information using the multiphoton pulses. We have just given a simple strategy for Eve to determine unambiguously the state sent by Alice, that works with some probability and provided that the pulse contains at least three photons. This is indeed a general result: Unambiguous discrimination between N states of a two-dimensional Hilbert space is only possible when at least $N-1$ copies of the state are available [26]. In this case, the N states $|\psi_i\rangle^{\otimes(N-1)}$ belong to the symmetric subspace of $(\mathbb{C}^2)^{\otimes(N-1)}$ of dimension N. Since the N states are always linearly independent (see Appendix B), unambiguous discrimination is possible. Above we have described a sequence of measurements allowing unambiguous discrimination between three copies of the four states $|\pm x\rangle, |\pm y\rangle$. The probability of success is given by the third measurement that discriminates between two quantum states having an overlap of $1/\sqrt{2}$, i.e., $p_{ok}=1-1/\sqrt{2}\sim 0.3$. However, better strategies should be expected if one acts globally on the three copies of the unknown state. For instance, one can use the natural generalization of the POVM described by Eqs. (12). For any $i=\pm x,\pm y$ one can define $|\psi_i^{\perp}\rangle \in (\mathbb{C}^2)^{\otimes 3}_{\mathrm{sym}}$ as the state orthogonal to the three vectors $|\psi_j\rangle^{\otimes 3}$, with $j\neq i$. Then, the searched measurement is given by the five positive operators summing up to the identity of $(\mathbb{C}^2)^{\otimes 3}_{\mathrm{sym}}$, denoted by $\mathbb{1}_{3,\mathrm{sym}}$,

$$\Pi_i=\frac{2}{3}|\psi_i^{\perp}\rangle\langle\psi_i^{\perp}|,$$

$$\Pi_?=\mathbb{1}_{3,\mathrm{sym}}-\sum_i \Pi_i. \tag{28}$$

The probability of having an inconclusive result is, where $|i^{(3)}\rangle=|i\rangle^{\otimes 3}$,

$$p_?=\langle i^{(3)}|\Pi_?|i^{(3)}\rangle=\frac{1}{2}. \tag{29}$$

Indeed, this measurement is optimal if we impose that the probability of conclusive result has to be the same for the four possibilities to be distinguished. In fact, from Chefles' work [27], we know that the maximal probability of unambiguous discrimination is equal to the reciprocal of the maximum eigenvalue of the operator

$$\frac{1}{4}\sum_{i=\pm x,\pm y}(|\psi_i^{\perp}\rangle\langle\psi_i^{\perp}|), \tag{30}$$

which gives $p_{ok}(3)=1/2$ [28]. Actually, Chefles' optimal discrimination method can be applied also to the case when the pulse contains $n>3$ photons. The optimal probability for discriminating between the four states under study knowing that n copies are available has been found numerically to be $p_{ok}(n)=1-(1/2)^{[(n-1)/2]}$, where $[\cdot]$ is the rounding to the closest lower integer.

The critical distance is given by the point at which Eve can block all the pulses containing less than three photons and those pulses with more than three photons for which the unambiguous discrimination has failed, that is, when Bob's raw rate reaches

$$R_{\mathrm{Bob}}=\sum_{n\geqslant 3} p_{ok}(n)p(n,\mu)\gtrsim\frac{1}{2}\sum_{n\geqslant 3} p(n,\mu). \tag{31}$$

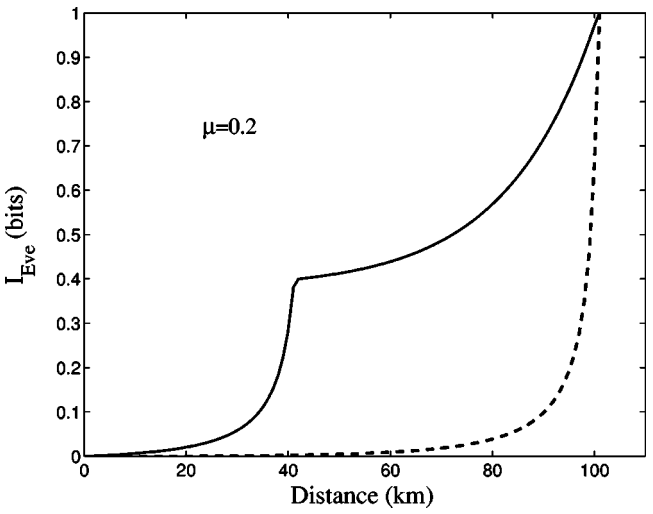

FIG. 4. The figure shows different eavesdropping attacks that take advantage of the presence of multiphoton pulses for the four-state protocol. The dashed line represents the attack where all pulses with less than three photons are blocked. Eve can however interpolate between different attacks as described in the text, depending on the channel losses. The solid line is Eve's information for this second possibility.

Eve's information is shown in Fig. 4, and the critical distance turns out to be of approximately 100 km [29]. Note that we take $\mu=0.2$, in order to make a fair comparison with BB84 using $\mu=0.1$. As for BB84 and for the same reason, the result also holds in very good approximation for finite detector efficiency η_d, provided that Eve cannot increase this efficiency.

We have just described an *intercept-resend strategy* that works well at large distances. For small distances however, this strategy is quite inefficient from Eve's point of view. Indeed, for those instances it is better for her to apply a different PNS attack, that we call *storing attack*: all single-photon pulses are blocked, while for all the multiphoton pulses, she keeps one photon in a quantum memory until the set reconciliation. Then, she has to distinguish between two nonorthogonal quantum states, say $|+x\rangle$ and $|+y\rangle$. She will apply the measurement maximizing her information obtaining [see Eq. (23)] $I_{\mathrm{Eve}}\sim 0.4$ and where the error probability is

$$p_e=\frac{1}{2}(1-\sqrt{1-|\langle +x|+y\rangle|^2})\sim 0.14. \tag{32}$$

Storing attacks are particularly dangerous as soon as there are errors in the transmission. If this is the case, the information Alice-Bob, I_{AB}, is smaller than one and indeed, it may be smaller than Eve's information (see Sec. IV for a more careful analysis). In a similar way to that described above,

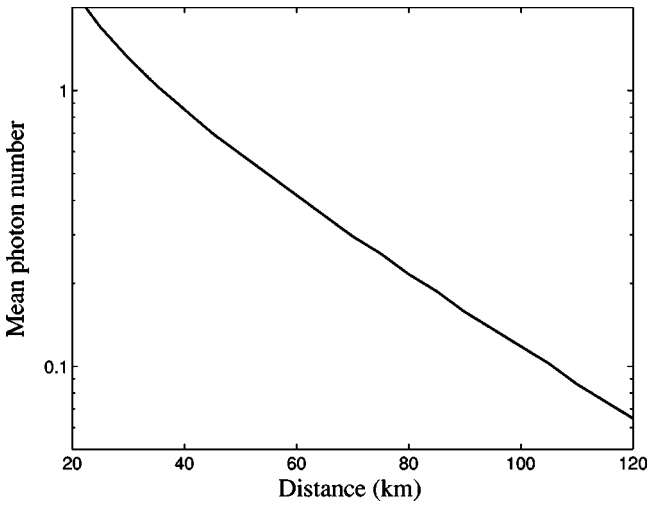

FIG. 5. The figure shows the mean photon number maximizing the key rate generation Eq. (33) as a function of the distance. For small distances one cannot take μ arbitrarily large, since the four states would become almost orthogonal and Eve could do an intercept-resend attack without being detected. For large distances, μ cannot be arbitrarily small, since the signal becomes negligible with respect to dark counts and the channel is completely noisy, $I_{AB}\sim 0$.

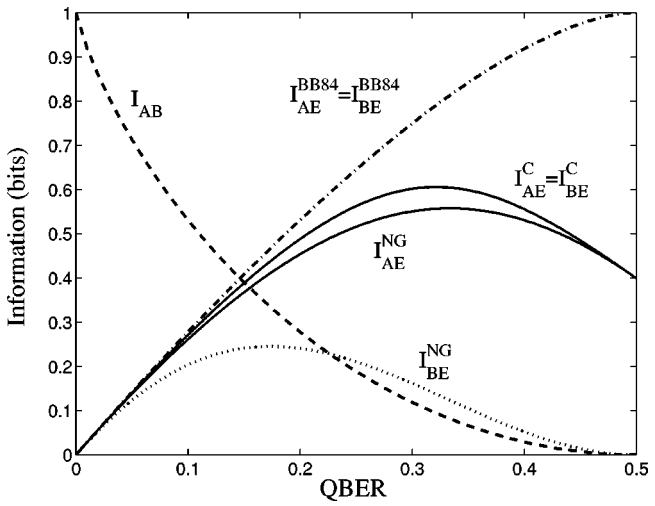

FIG. 6. The figure shows Alice's and Bob's versus Eve's information for individual attacks using the cloning machines introduced by Cerf and by Niu and Griffiths. The curve for the standard BB84 scheme is included for comparison.

depending on the channel losses, Eve can interpolate between the storage and the intercept-resend attacks. The corresponding information curves are shown in Fig. 4.

The presence of multiphoton pulses represents a serious drawback, since Eve can take advantage of them for acquiring information on the sent bit. Since we do not consider advantage distillation protocols, the honest parties can extract a key when Eq. (2) is satisfied. This means that the secret bit rate generation, after error correction and privacy amplification, is

$$R_{\text{key}}=\frac{1}{4}R_{\text{Bob}}(1-I_{\text{Eve}}), \tag{33}$$

where R_{Bob} is the raw rate of Eq. (4). The 1/4 term takes into account the set reconciliation process (Bob has to choose the right measurement and obtain the right outcome), and the last term comes from the privacy amplification protocol. Note that we assume for simplicity no errors between Alice and Bob, $I_{\text{AB}}=1$.

There is in principle an obvious way of avoiding the influence of multiphoton pulses: to decrease the pulse mean photon number. Nevertheless, this solution may be very inefficient, since the raw rate R_{Bob} is approximately proportional to μ. Therefore, there is a compromise from the point of view of key generation. Using the same techniques as for Fig. 4, for any δ one can compute the optimal μ maximizing R_{key}. The corresponding curve is shown in Fig. 5. Note that mean photon numbers $\sim$0.2 are indeed optimal for losses $\sim$20 dB.

C. Individual attacks using cloning machines

All the eavesdropping strategies studied up to now take advantage of the fact that the technological power for the honest parties has some limitations. In particular, Eve uses the multiphoton pulses for acquiring information on the sent bit without introducing any error. Nevertheless, the present protocol must also be analyzed under the presence of errors, even at the single-photon level. It may happen that a small amount of error would allow Eve to gain a large amount of information making the protocol unpractical. Indeed, these are the attacks Eve would apply at very short distances, where she cannot block almost any pulse and almost all the nonempty pulses reaching Bob contain just one photon.

The optimal individual eavesdropping strategy for this protocol is unknown. Nevertheless, note that the quantum structure is the same as for the BB84 scheme, so it seems natural to consider its robustness against attacks using asymmetric phase covariant cloning machines [30,31]. These machines, that are briefly described in Appendix C, clone in an optimal way all the states in a plane of the Bloch sphere. Let us stress here that they provide the optimal eavesdropping for the BB84 protocol [32]. The action of these machines in the protocol is depicted in Fig. 6.

Key distillation using privacy amplification is possible whenever Eq. (2) is fulfilled. This means that the honest parties can tolerate an error up to $\sim$15%, slightly larger than the 14.67% for the BB84. There are two facts in these curves that deserve explanation. First, note that the Cerf cloning machine [30] is clearly more efficient from Eve's point of view than the Niu-Griffiths one [31]. Second, note the surprising decreasing behavior of Eve's information for large values of the quantum bit error rate (QBER). Both of them are related to the quantum correlations introduced by each of the cloning machines between Eve and Bob, and the sifting procedure used in the described protocol.

Eve waits until the sifting process before doing her measurement. If, for instance, Alice announces $|+x\rangle$, $|+y\rangle$ and Bob accepts the symbol, Eve knows that Bob has successfully projected onto either $|-x\rangle$ or $|-y\rangle$. Then, she modifies her quantum state according to this information. The fact that

Bob has got a conclusive result (he could discriminate between the two nonorthogonal states) increases also the distinguishability on Eve's side because of the quantum correlations. On the one hand, this justifies why the Cerf cloning machine is more efficient for eavesdropping. It establishes stronger correlations between Eve and Bob, and this helps Eve after the sifting process. On the other hand, this also explains the decreasing behavior of Eve's information curves large QBER. For very large disturbances, the correlations between Eve and Bob decreased, and knowing that Bob has obtained a conclusive result does not help her too much. Thus, it is better to keep some quantum correlations with Bob, in such a way that his successful unambiguous discrimination increases the distinguishability on Eve's side too. In the limiting case of maximum error, Eve just takes the state sent by Alice and prepares at random one of the four possible states for Bob (or in equivalent terms, she forwards a completely noisy state). Her information is simply given by Eq. (23) as expected.

D. PNS+cloning attacks

The eavesdropping strategies analyzed up to now take advantage, either of the presence of multiphoton pulses (PNS attacks) or of the errors on Bob's side (cloning attacks). However for losses such that Eve can simulate the expected rate even if she blocks all the single-photon pulses, she can combine the two type of attacks, if she is allowed to introduce some errors. This basically corresponds to distances $\ell \gtrsim 40$ km (see Fig. 4). There, Eve counts the number of photons in the pulse and stops those having one photon. For all the two-photon pulses, she applies an asymmetric phase covariant $2\rightarrow 3$ cloning machine, and forwards one of the clones to Bob. This operation introduces errors, depending on the quality of Bob's clone. In general, for a pulse having n photons, she uses an $n\rightarrow n+1$ cloning machine, although in this section we consider only the $2\rightarrow 3$ case, since p_2 is significantly larger than p_3. As far as we know this type of attack has been never considered before, nor have the corresponding phase covariant $n\rightarrow m$ asymmetric cloning machines. In Appendix D we describe two unitary transformations generalizing, in a nonoptimal way, the asymmetric $1\rightarrow 2$ cloning machines to the $2\rightarrow 3$ case [33].

The attack goes as follows. Eve counts the number of photons in the pulse. All the single-photon pulses are blocked, while for those pulses having two photons she applies one of the $2\rightarrow 3$ cloning machines shown in Appendix D. In this case it is unclear which of the clone states she has to forward to Bob. It turns out that for small disturbances, such that Eve's information is smaller than I_{AB}, there is almost no difference between the two cases. Figure 7 shows the information Eve can get with this strategy as a function of the disturbance on Bob's side. We consider that Bob receives one of the two clones with the same fidelity, i.e., either the first or the second qubit of Eqs. (D3) or (D5). Key distillation is possible using error correction and one-way privacy amplification up to disturbances of approximately 8.5%.

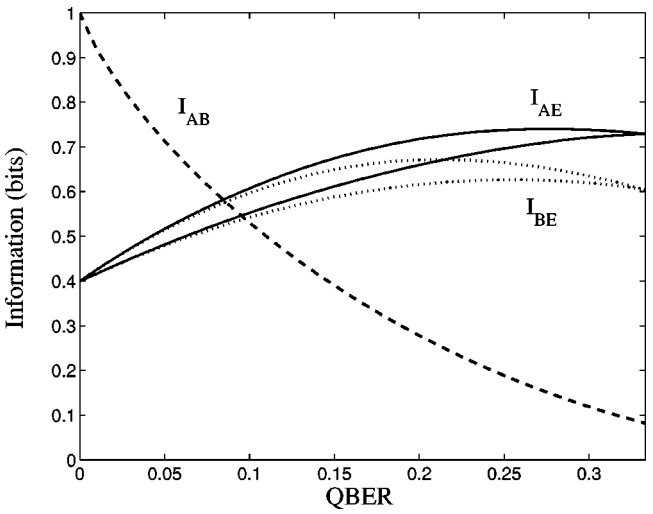

FIG. 7. The figure shows Alice's and Bob's versus Eve's information for attacks using the cloning machines described in Appendix D. Upper curves correspond to the cloning machine of Eq. (D3), which is more powerful from Eve's point of view.

E. Geneva-Lausanne experiment

The four-state protocol is at the level of state preparations and measurements, identical to the BB84 scheme. It only differs in the sifting process, less efficient in the absence of Eve by a factor of two on the raw key, but more robust against PNS attacks. Thus, all the existing experimental implementations of the BB84 protocol can be thought of as implementations of the new four-state protocol.

Let us analyze the recent Geneva-Lausanne experiment [15], where a key was distributed over 67 km using the BB84 scheme. The mean photon number of the pulses used in this experiment was indeed 0.2 photons/pulse, so all our results directly apply. According to Fig. 1, the protocol is not secure at this distance because of the PNS attack, even for $\mu=0.1$ (and BB84 encoding). However this is not the case if one uses the new protocol. The experimental QBER was approximately 5%, where 4% was due to dark counts on the detector and 1% due to optical imperfections. As said above, Eve is assumed to have only access to the optical error. Then $I_{\mathrm{AB}}=I(0.05)\sim 0.71$ bits, while I_{Eve} (see Fig. 7) is clearly smaller than 0.5. Thus, Alice and Bob can safely distill a key. Note that even in the more restrictive scenario where Eve can take advantage of the full error (including the detector noise), her information is smaller than I_{AB} and the protocol is secure. Therefore, this implementation becomes secure against the PNS attacks considered in this work just by changing the sifting process.

IV. GENERALIZATION TO MORE SETS

The detailed analysis of the four-state protocol has given us insight into the way of designing QKD protocols resistant to PNS attacks. The presence of multiphoton pulses is still a problem, since they open the possibility of unambiguous discrimination or storing attacks providing Eve with full or partial information. But there is a simple way of improving the robustness of the protocol: just adding more states for the

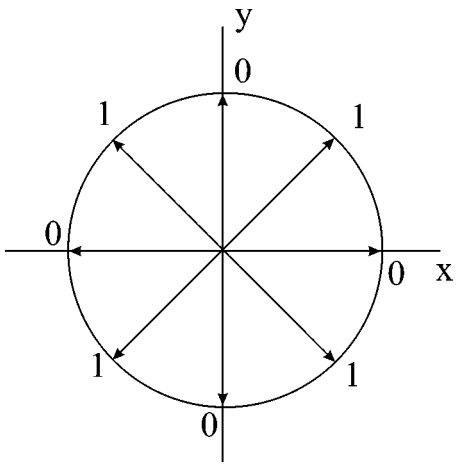

FIG. 8. Bit encoding in a protocol using four bases.

encoding. A quite natural generalization of the previous protocol follows this idea and consists of adding more bases in a plane of the Bloch sphere for the encoding of the bit, as shown in Fig. 8 for the case of four bases (eight states). On the one hand more photons (or copies of the unknown state) are needed for the unambiguous discrimination to be possible. On the other hand the overlap between the two announced states decreases, which is also good against storing attacks. Nevertheless, the key rate decreases unless we use a larger mean photon number, which increases the presence of multiphoton pulses, that are dangerous for the security. Thus, a compromise appears. The aim of this section is to explore this fact by analyzing the resistance of this generalized protocols against the two type of attacks mentioned above: PNS with unambiguous discrimination and storing attacks.

Any protocol is uniquely defined by the number of bases n_b used for the bit encoding. We will not consider a very large number of bases, since the protocol would become impractical. In the previous sections we had $n_b=2$, while Fig. 8 depicts the case $n_b=4$. If Alice wants to send a bit x, she chooses at random between the n_b states encoding x and sends it to Bob. Bob measures at random in any of the n_b bases. Then, Alice announces the sent state plus, again randomly, one of the two neighboring states (encoding $1-x$). Bob accepts the bit when (i) he has measured in one of the two bases associated to the two states announced by Alice and (ii) his measurement outcome is orthogonal to one of these states. Indeed, this allows him to discard one of the two possibilities and to infer x. Thus, Bob needs to choose the right measurement and obtain the right outcome, which happens with probability

$$p_x=\frac{1}{n_b}\sin^2\left(\frac{\pi}{2n_b}\right). \tag{34}$$

As usual, in order to make a fair comparison, we impose for any protocol that at very large distances (attenuations) the raw rate is the same as in the standard BB84 with $\mu=0.1$. This implies that

$$\mu(n_b)=\frac{1}{20p_x}=\frac{n_b}{20\sin^2\left(\frac{\pi}{2n_b}\right)}. \tag{35}$$

Note that for large n_b, $\mu(n_b)\sim n_b^3$. This means that the mean photon number becomes significant when n_b increases and we are not longer dealing with weak pulses.

Eve has now to discriminate between $2n_b$ one-qubit states, and this can be done with certainty only when $n_e=2n_b-1$ copies of the unknown state are available (see Ref. [26] and Appendix B). The maximum probability of success, p_{ok}, correspond to the maximum eigenvalue of the operator [27]

$$\frac{1}{2n_b}\sum_{k=0}^{n_e}|k^\perp\rangle\langle k^\perp|. \tag{36}$$

Here $|k^\perp\rangle$ denotes the state in $(\mathbb{C}^2)_{\rm sym}^{\otimes n_e}$ orthogonal to all $|j\rangle^{\otimes n_e}$, where $j=0,...,n_e$ but $j\neq k$ and

$$|k\rangle=\frac{1}{\sqrt{2}}\begin{pmatrix}1\\ e^{ik\pi/n_b}\end{pmatrix}. \tag{37}$$

We have numerically calculated these probabilities up to $n_b=8$ and they appear to be given by the formula $p_{ok}(n_b)=n_b/4^{n_b-1}$, although we do not have an analytical proof. The critical attenuation δ_1 (in decibels) where the protocol ceases to be secure against this attack has to be such that Eve can simulate the expected rate by the number of pulses containing at least n_e photons and giving a conclusive result. This leads to

$$\sum_{n>0}p[n,\mu(n_b)10^{-\delta_1/10}][1-(1-\eta_d)^n]$$

$$=p_{ok}(n_b)\sum_{m\geqslant n_e}p[m,\mu(n_b)][1-(1-\eta_d)^{(m-n_e+1)}]. \tag{38}$$

The corresponding curve is shown in Fig. 9.

There are other attacks, exploiting the presence of multiphoton pulses, that provide Eve with partial information without introducing errors. For instance, Eve can count the number of photons and keep n_s of them, depending on the channel attenuation, without being detected. She waits until the basis reconciliation and performs the measurement maximizing her information [see Eq. (23)]. These attacks can be very dangerous as soon as we consider errors on the transmission. We assume that the main sources of errors are the detector noise, quantified by the probability p_d of having a dark count, and the optical error $E_{\rm opt}$. The total error E for a channel attenuation of δ is approximately equal to

$$E=\frac{p_d/2}{p_d+\mu(n_b)\eta_d t}+E_{\rm opt}, \tag{39}$$

since half of the dark counts produce a click in the wrong detector. Thus, for any distance one can compute the amount

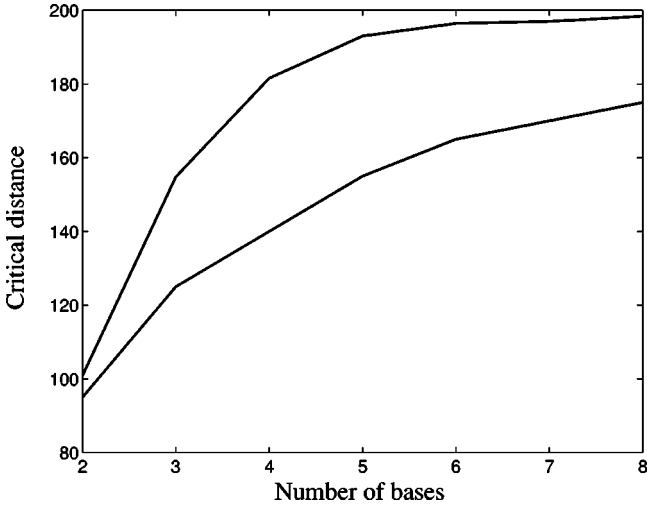

FIG. 9. Critical distance for protocols using n_b bases. Upper curve is given by PNS attacks using unambiguous discrimination, while the lower curve corresponds to storing attacks, as explained in the text. Storing attacks are clearly more efficient from Eve's point of view.

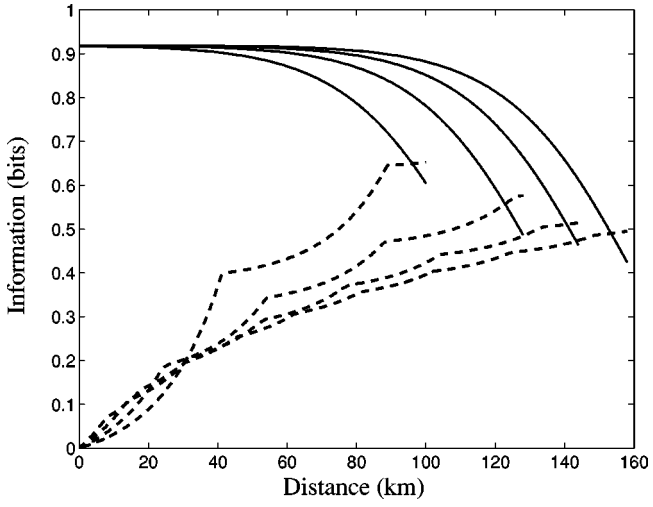

FIG. 10. Information curves as a function of the distance for protocols using $n_b=2,...,5$ bases. Solid lines represent the information Alice-Bob: At large distances, the signal level is small compared to dark counts and the QBER becomes important [see Eq. (39)]. Dashed lines show Eve's information: At large distances, she can keep many photons without being detected, acquiring more information on the sent state. The point where the two curves cross defines the critical distance where the protocol is no longer secure.

of errors and the corresponding $I_{\mathrm{AB}}=I(E)$. If I_{Eve} is larger than I_{AB}, the protocol is not secure. For any number of stored photons n_s, we can define a critical attenuation such that the honest parties cannot notice Eve's storing attack. This attenuation corresponds to the point where

$$\sum_{n>0} p[n,\mu(n_b)10^{-\delta(n_s)/10}][1-(1-\eta_d)^n]$$

$$=\sum_{m\geqslant n_s} p[m,\mu(n_b)][1-(1-\eta_d)^{(m-n_s)}]. \qquad (40)$$

For intermediate attenuations (distances), Eve can interpolate between two attacks, as described above. In this way, we can compute the two curves I_{AB} and I_{Eve} as a function of the distance. Figure 10 shows the obtained results, where we took $\eta_{\mathrm{det}}=0.1$, $p_d=10^{-5}$, and $E_{\mathrm{opt}}=1\%$. The point where $I_{\mathrm{AB}}=I_{\mathrm{Eve}}$ provides the critical distance δ_2 for this type of attacks. In Fig. 9 we plot both the δ_1 and δ_2 as a function of n_b. It is quite plausible that $\min(\delta_1,\delta_2)$ gives a good estimation for δ_c, the critical distance associated to the unknown optimal attack. Thus, one can safely conclude that a key can be established using a reasonable number of bases up to distances of the order of 150 km [29,35].

V. CONCLUSIONS

Unconditional security of quantum cryptography relies on some experimental assumptions that are not practical with present-day technology. Thus, in a more realistic scenario, the honest parties have to deal with approximated single-photon sources, noisy channels, inefficient detectors, and so on, while no limitation on the eavesdropper technology should be assumed. This opens the possibility for alternative eavesdropping attacks, taking advantage of Alice and Bob's technological imperfections. Indeed, using as a reference the BB84 scheme with $\mu=0.1$, all the known protocols become insecure against PNS attacks for channel losses of the order of 13 dB.

In this paper, we show how to construct QKD protocols resistant against a class of PNS attacks up to channel losses of 40 dB. There are two possibilities for that: (i) to exploit the nonorthogonality of quantum states in a different way, as in the presented four-state protocol or (ii) to include a strong reference pulse that must be always detected by Bob. Both possibilities seem achievable with current technology. In the first case, already existent implementations of the BB84 protocol [15] provide an experimental demonstration of QKD secure against PNS attacks, when the alternative sifting process of the new four-state protocol is applied. Moreover it suggests a connection between discrete and continuous variables QKD schemes in the limit of a large number of bases, $n_b\rightarrow\infty$, that deserves further investigation.

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APPENDIX A

In this appendix we show that the overlap between all the states in Fig. 3 cannot be decreased by the same quantum operation. Using the parametrization of Eq. (26), one can see that

$$|0_b\rangle=c|0_a\rangle+c'|1_a\rangle,$$

$$|1_b\rangle = c'|0_a\rangle + c|1_a\rangle, \tag{A1}$$

where

$$c = -\frac{\cos\eta}{\sin\eta} \quad c' = \frac{1}{\sin\eta}. \tag{A2}$$

Now, consider a quantum operation M mapping with some probability p_a the states in set a into some new states, $|0'_a\rangle$ and $|1'_a\rangle$, such that $\langle 0'_a|1'_a\rangle = 0$. This means that

$$M|i_a\rangle = \frac{1}{\sqrt{p_a}}|i'_a\rangle, \tag{A3}$$

where $i = 0,1$. Because of the linearity of quantum mechanics, the states in set b will be mapped into

$$|0'_b\rangle = \frac{1}{\sqrt{p_b}}(c|0'_a\rangle + c'|1'_a\rangle),$$

$$|1'_b\rangle = \frac{1}{\sqrt{p_b}}(c'|0'_a\rangle + c|1'_a\rangle), \tag{A4}$$

with probability

$$p_b = \frac{1+\cos^2\eta}{\sin^2\eta}\frac{1}{p_a}. \tag{A5}$$

Their overlap is

$$|\langle 0'_b|1'_b\rangle| = \frac{2\cos\eta}{1+\cos^2\eta} \geqslant \cos\eta, \tag{A6}$$

i.e., the states in set b become less distinguishable.

APPENDIX B

In this appendix we will show that $N-1$ copies of N one-qubit state are always linearly independent (see, also Ref. [26]). Consider $N-1$ copies of $N-1$ general states of one qubit, $|\psi_i\rangle$ with $i = 1,...,N-1$. They belong to the symmetric subspace $(\mathbb{C}^2)^{\otimes(N-1)}_{\text{sym}}$ of dimension N. Our aim is to add a new state and see when this state can be written as a linear combination of the previous ones. In other terms, we want to find a state $|\psi_N\rangle \in \mathbb{C}^2$ such that the determinant of the $N \times N$ matrix

$$(|\psi_1\rangle^{\otimes(N-1)} \cdots |\psi_{N-1}\rangle^{\otimes(N-1)} |\psi_N\rangle^{\otimes(N-1)}) \tag{B1}$$

is zero. Note that the norm of the state does not play any role, so we can write

$$|\psi_N\rangle = \begin{pmatrix} 1 \\ x \end{pmatrix}, \tag{B2}$$

where x is an unbounded complex number. Condition (B1) then gives an $N-1$ degree polynomial equation on x. There are $N-1$ solutions, that correspond to the $N-1$ trivial cases $|\psi_N\rangle = |\psi_i\rangle$ for $i = 1,...,N-1$. Thus, $N-1$ copies of any N different one-qubit state are always linearly independent.

APPENDIX C

In this appendix we briefly describe the asymmetric phase covariant cloning machines introduced in Refs. [30] and [31]. These machine clone with maximal fidelity all the states that lie in the a plane of the Bloch sphere, say xy. At first sight, their only difference is that the one in Ref. [30] uses as an input state a two-qubit reference state plus the state to be cloned, while for the second machine, one qubit suffices as ancillary system.

Consider an input state to be cloned, and a one-qubit ancillary system in a reference state, say $|0\rangle$. The Niu-Griffiths cloning machine [31] is defined by the following unitary transformation:

$$U_{12}^{\text{NG}}|00\rangle_{12} = |00\rangle$$

$$U_{12}^{\text{NG}}|10\rangle_{12} = \cos\gamma|10\rangle + \sin\gamma|01\rangle, \tag{C1}$$

with $0 \leqslant \gamma \leqslant \pi/2$. From the definition it is evident that this transformation does not affect in the same way the two poles $|\pm z\rangle$ of the Bloch sphere. Nevertheless, this is not the case for those state lying in the xy plane, i.e., $|\vartheta\rangle = (|0\rangle + e^{i\vartheta}|1\rangle)/\sqrt{2}$. The searched clones are the mixed local states resulting from tracing either the first or the second qubit on the state resulting from the application of Eq. (C1)

$$\rho_i = tr_{2-i}[\Pi_{\text{NG}}(\vartheta)], \tag{C2}$$

where $i = 1,2$ and $\Pi_{\text{NG}}(\vartheta)$ is the projector onto $U_{\text{NG}}|\vartheta\rangle|0\rangle$. One can easily see that $\forall\vartheta$

$$\rho_1 = \cos\gamma|\vartheta\rangle\langle\vartheta| + (1-\cos\gamma)\frac{\mathbb{1}}{2}$$

$$\rho_2 = \sin\gamma|\vartheta\rangle\langle\vartheta| + (1-\sin\gamma)\frac{\mathbb{1}}{2}. \tag{C3}$$

Then, the corresponding clone fidelities, defined as $F_i = \langle\vartheta|\rho_i|\vartheta\rangle$, are $(1+\cos\gamma)/2$ and $(1+\sin\gamma)/2$. The larger the fidelity for the first clone, the smaller for the second. Equality is achieved when $\cos\gamma = \sin\gamma$, and then $F_1 = F_2 = (1+1/\sqrt{2})/2$.

The second type of cloning machine we consider are those introduced in Ref. [30]. There, two qubits are used as the ancillary system, and the unitary transformation is, for any input state $|\psi\rangle \in \mathbb{C}^2$

$$U_{12}^{C}|\psi\rangle|00\rangle = F|\psi\rangle|\Phi^+\rangle + (1-F)\sigma_z|\psi\rangle|\Phi^-\rangle + \sqrt{F(1-F)} \times (\sigma_x|\psi\rangle|\Psi^+\rangle + i\sigma_y|\psi\rangle|\Psi^-\rangle), \tag{C4}$$

where

$$|\Psi^\pm\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle),$$

$$|\Psi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle), \tag{C5}$$

define the standard Bell basis. It is not difficult to see that the local state in the first two qubits is the same as in Eq. (C2) if one takes $F=(1+\cos\gamma)/2$.

Eve can use these transformations in order to obtain information about the sent bit. She clones the state sent by Alice, and she forwards the first clone to Bob and keeps the second. Obviously there is a compromise between the quality of the two clones: The better Eve's clone the worse Bob's state. Or in other words, the more the information intercepted by Eve, the more the errors on Bob's side, that allow the honest parties to detect Eve's intervention. As seen above, the two machines are in many senses equivalent (especially as far as for the cloning fidelities are concerned). However the two attacks differ in the amount of correlations Eve establishes with Bob. This fact is going to be very important for the type of protocols analyzed in this work.

APPENDIX D

In this appendix we give two different unitary transformations that somehow generalizes the $1\rightarrow 2$ asymmetric cloning machines to the $2\rightarrow 3$ case.

The first machine is mainly inspired by Niu-Griffiths construction. The initial input state corresponds to two copies of an unknown one-qubit state, $|\psi\rangle^{\otimes 2}\in(\mathbb{C}^2\otimes\mathbb{C}^2)_{\rm sym}$. Using a two-dimensional ancillary system, say in state $|0\rangle$, one can define the unitary operation

$$U_{23}^{\rm NG}|00\rangle|0\rangle = |000\rangle,$$

$$U_{23}^{\rm NG}|\Psi^{+}\rangle|0\rangle = \frac{\cos\gamma(|010\rangle+|100\rangle)+\sin\gamma|001\rangle}{\sqrt{1+\cos^2\gamma}},$$

$$U_{23}^{\rm NG}|11\rangle|0\rangle = \frac{\cos\gamma|110\rangle+\sin\gamma(|011\rangle+|101\rangle)}{\sqrt{1+\sin^2\gamma}}. \tag{D1}$$

As in the $1\rightarrow 2$ case, this machine has not the same effect on the states $|0\rangle$ and $|1\rangle$. After some lengthy algebra one can see that all the states $|\psi\rangle$ in the xy plane are cloned with the same fidelities, that are equal to (see also Fig. 11)

$$F_1^{\rm NG}=F_2^{\rm NG}=\frac{1}{2}+\frac{\cos\gamma}{2\sqrt{3+\cos(2\gamma)}}+\frac{1}{\sqrt{17-\cos(4\gamma)}},$$

$$F_3^{\rm NG}=\frac{1}{2}+\frac{\sin\gamma}{2\sqrt{3+\cos(2\gamma)}}+\frac{\sin(2\gamma)}{\sqrt{17-\cos(4\gamma)}}. \tag{D2}$$

Note that when $\gamma=\pi/4$, $F_1^{\rm NG}=F_3^{\rm NG}=(6+2\sqrt{2}+\sqrt{6})/12\sim 0.94$, slightly larger than the fidelity of the $2\rightarrow 3$ universal symmetric cloning machine of Ref. [34]. It has to be stressed that the fidelity for the third clone never reaches the value of one, contrary to what happens for the $1\rightarrow 2$ case. As we learned from the analysis of individual attacks, in our protocols it is more convenient to Eve to introduce an extra ancillary system, in such a way that she is better correlated to Bob's result. This can be done introducing an ancillary system on Eve's side, such that the action on the states of the computational basis is symmetrized. Note that in the $1\rightarrow 2$ case this procedure allows to pass from the Niu-Griffiths to the Cerf cloning machine. The resulting machine can be expressed as

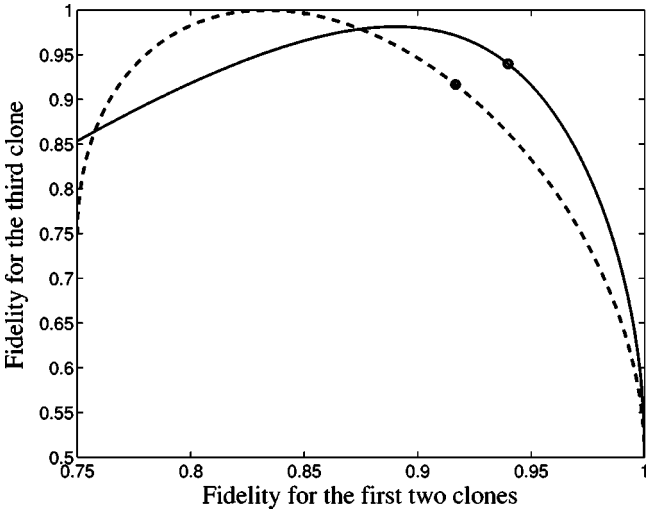

FIG. 11. Cloning fidelities for the $2\rightarrow 3$ cloning machines defined by Eqs. (D4) (solid line) and (D5) (dashed line). The circles correspond to the points where the cloning fidelities are equal.

$$U_{23}^{\rm NGs}|s\rangle|00\rangle=(U_{23}^{\rm NG}|s\rangle|0\rangle)|0\rangle+(\tilde{U}_{23}^{\rm NG}|s\rangle|0\rangle)|1\rangle, \tag{D3}$$

where $|s\rangle=|00\rangle$, $|\Psi^{+}\rangle$, $|11\rangle$ and $\tilde{U}_{23}^{\rm NG}$ has the same form as $U_{23}^{\rm NG}$ but interchanging zeros and ones, i.e.,

$$\tilde{U}_{23}^{\rm NG}|00\rangle|0\rangle=\frac{\cos\gamma|001\rangle+\sin\gamma(|100\rangle+|010\rangle)}{\sqrt{1+\sin^2\gamma}},$$

$$\tilde{U}_{23}^{\rm NG}|\Psi^{+}\rangle|0\rangle=\frac{\cos\gamma(|101\rangle+|011\rangle)+\sin\gamma|110\rangle}{\sqrt{1+\cos^2\gamma}},$$

$$\tilde{U}_{23}^{\rm NG}|11\rangle|0\rangle=|111\rangle. \tag{D4}$$

The local state of each of the three first qubits is a combination of the identity with the initial pure state as expected. The cloning fidelities are again equal to Eq. (D2).

The second machine we consider is based on Cerf construction [30]. As an input state we have two qubits of an unknown one-qubit state plus a two-qubit ancillary system. Then, we define the following unitary operation:

$$U_{23}^{C}|\psi\rangle^{\otimes 2}|00\rangle=v|\psi\rangle^{\otimes 2}|\Phi^{+}\rangle+x(\tilde{\sigma}_z|\psi\rangle^{\otimes 2}|\Phi^{-}\rangle + \tilde{\sigma}_x|\psi\rangle^{\otimes 2}|\Psi^{+}\rangle+i\tilde{\sigma}_y|\psi\rangle^{\otimes 2}|\Psi^{-}\rangle), \tag{D5}$$

where, for $k=x,y,z$,

$$\tilde{\sigma}_k=\sigma_k\otimes 1+1\otimes\sigma_k, \tag{D6}$$

and $v^2+8x^2=1$. One can see that for any input state in the Bloch sphere, the local state of the first two qubits are two identical clones with fidelity $F_1^C=F_2^C=1-2x^2$, while in the third qubit we have another clone with fidelity $F_3^C=1-(v-3x)^2/2$. Thus, the machine (D5) is an asymmetric universal cloning machine, i.e., not phase covariant. Indeed, at the point where the three fidelities are equal, we recover the $2\rightarrow3$ cloning fidelity of Ref. [34] $F_1^C=F_3^C=11/12$ (see also Fig. 11). Note also that in this case, F_3^C can be equal to one. Moreover, there are some points where, for a given fidelity for the first two clones, the fidelity for the third one is larger using this cloning machine than for the phase covariant machine of Eq. (D4). This shows that the latter is not the optimal phase covariant asymmetric $2\rightarrow3$ cloning machine. One is tempted to generalize Cerf construction in a direct way, defining a phase covariant machine by changing the coefficient of one of the error terms in Ref. [D5]. However, we found that the resulting operation is not unitary. Therefore, we can only propose two possible asymmetric phase covariant machines, although we know that they are not optimal. Nevertheless, it is quite reasonable to suppose that the increase on Eve's information will not be very significant when using the, at present unknown, optimal machine [33].

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